

JEE: OSCILLATIONS

- 1) Two particles A & B are connected by a light inextensible string passing over a frictionless pulley. A is attached to a spring of force constant K fixed to a wall and B hangs freely. The system is released from rest after stretching the spring by a small amount. Find the time period of oscillation of a system.

ANSWER

On A $\Rightarrow$ Spring force = $-Kx$
On B $\Rightarrow$ Weight = $m_B g$

$$\ddot{x} = \frac{-K}{m_A + m_B} x$$

For A,
 $m_A \ddot{x} = -Kx - T$ — (1)

$$\Rightarrow \omega = \sqrt{\frac{K}{m_A + m_B}}$$

For B,
 $m_B \ddot{x} = T$ — (2)

(1) + (2)

$$\text{ANS} \Rightarrow T = 2\pi \sqrt{\frac{m_A + m_B}{K}}$$

$$(m_A + m_B) \ddot{x} = -Kx$$

- 2) A particle moves along x axis under a force such that its acceleration is given by

$$a = -\omega^2 x + \lambda x^3 \quad [\lambda \text{ is very small}]$$

Determine whether the motion is simple harmonic, if not, find time period

ANSWER

For SHM, $a \propto -x$

But here extra term λx^3 exists $\rightarrow$ non linear

$\Rightarrow$ Motion is not pure SHM

If x is small $\Rightarrow x^3 \ll x$

So:

$$a \approx -\omega^2 x$$

motion $\approx$ SHM for small amplitude

3. A block of mass m is attached to a spring and placed on a smooth horizontal surface. A bullet of mass m moving with velocity v strikes the block and gets embedded in it

Find.

A) amplitude

B) Max compression

C) Time Period

ANSWER

Initial momentum, mv

Final mass = $2m$, velocity = v'

$$mv = 2m \cdot v' \Rightarrow v' = \frac{v}{2} \quad \text{--- (2)}$$

$$\omega = \sqrt{\frac{k}{2m}} \quad \text{--- (1)}$$

$$A = \frac{v'}{\omega}$$

$$A = \frac{v/2}{\sqrt{k/2m}}$$

$$A = \frac{v}{\sqrt{2}} \sqrt{\frac{m}{k}}$$

$$x_{\max} = A = \frac{v}{\sqrt{2}} \sqrt{\frac{m}{k}}$$

$$T = 2\pi \sqrt{\frac{2m}{k}}$$

Q. A simple pendulum of string length 30cm performs 20 oscillations in 10s. The length of the string required for the pendulum to perform 40 oscillations in the same time duration is? _____ cm

Sol: We know

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$T \propto \sqrt{L} \Rightarrow T^2 \propto L$$

Case-1

$$L_1 = 30\text{cm}$$

$N_1 = 20$ is the no. of oscillations completed in total time $t_1 = 10\text{sec}$

$$T_1 = \frac{\text{Total time}}{\text{no. of oscillation}} = \frac{10}{20} = 0.5\text{sec}$$

Case-2

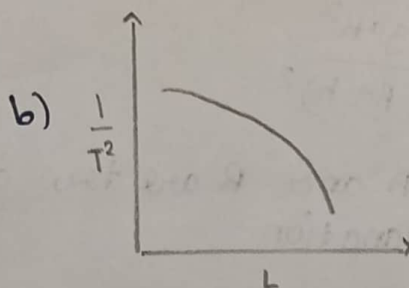
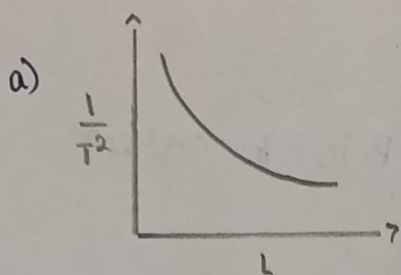
$$N_2 = 40 \Rightarrow t_2 = t_1 = 10\text{sec}$$

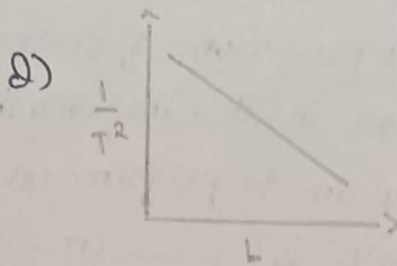
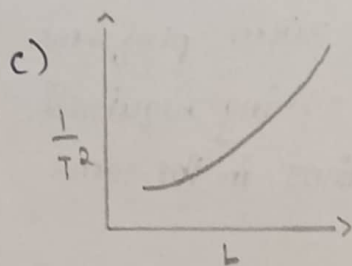
$$T_2 = \frac{40}{10} = 0.25\text{sec}$$

$$\frac{L_2}{L_1} = \left(\frac{T_2}{T_1}\right)^2 \Rightarrow \frac{L_2}{30} = \left(\frac{0.25}{0.50}\right)^2 \Rightarrow \frac{L_2}{30} = \frac{1}{4} \Rightarrow L_2 = 7.5\text{cm}$$

$\therefore$ The required length of the string is 7.5cm

Q. Using simple pendulum experiment g is determined by measuring its time period T . Which of the following plots represent the correct relation between pendulum length L and time period T ?





Sol: $T = 2\pi \sqrt{\frac{l}{g}}$

$$T^2 = \frac{4\pi^2 L}{g}$$

$$\frac{1}{T^2} = \left(\frac{g}{4\pi^2}\right) \times \frac{1}{L}$$

'g' and $4\pi^2$ are constants, $\therefore \frac{g}{4\pi^2} = k$

$$\Rightarrow y = kx$$

where $y = \frac{1}{T^2}$ and $x = L$

The mathematical equation $y = kx$ represents rectangular hyperbola where $\frac{1}{T^2} \rightarrow 0$ if $L \rightarrow \infty$, x-axis is horizontal asymptote also $\frac{1}{T^2} \rightarrow \infty$ if $L \rightarrow 0$, $\therefore$ resulting graph is option a)

Q Assertion: Time period of a simple pendulum is longer at the top of a mountain than that at the base of the mountain

Reason: Time period of a simple pendulum decreases with increasing value of acceleration due to gravity and vice versa

Sol: As h increases, g decreases, T increases

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$g = \frac{g_0 R^2}{(R+h)^2}$$

$\therefore$ Both A and R are true and R is the correct explanation

JEE Questions Oscillations

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①

Q. 1) A spring of force constant 15 N/m is cut into two pieces. If the ratio of their length is $1:3$, then the force constant of smaller piece is --- N/m . [JEE Main 2026]

- (A) 20 (B) 45
(C) 60 (D) 15

Ans: (*) For uniform spring;

$$k \propto \frac{1}{L}$$

$$K = 15 \text{ N/m}$$

$$\frac{L_1}{L_2} = \frac{1}{1+3}$$

$$L_1 = \frac{L_2}{4} \quad \text{--- (1)}$$

$$\text{So; } k_1 = K_2 \frac{L_2}{L_1} \Rightarrow 15 \times \frac{L}{\frac{L}{4}}$$

$$\Rightarrow 15 \cdot 4 = 60 \text{ N/m} \quad \text{(C)}$$

Q. 2) A simple pendulum of string length 30 cm performs 20 oscillations in 10 sec . The length of the string required for the pendulum to perform 40 oscillations in the same time duration is --- cm . [Assume mass remains same]. [JEE Main 2026]

Ans: * We know;

$$T_1 = \frac{\text{Total time}}{\text{No. of oscillations}} \Rightarrow \frac{10}{20} \Rightarrow 0.5 \text{ sec}$$

(2)

$$T_2 = \frac{10}{40} \Rightarrow 0.25 \text{ sec}$$

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \{\text{Formula}\}$$

We can derive that;

$$\frac{T_2^2}{T_1^2} = \frac{L_2}{L_1}$$

$$L_2 = \left(\frac{T_2}{T_1}\right)^2 \times L_1$$

$$L_2 \Rightarrow 30 \left(\frac{0.25}{0.5}\right)^2$$

$$L_2 \Rightarrow 30 \times \frac{1}{4} \Rightarrow \frac{15}{2} \Rightarrow 7.5 \text{ cm}$$

$$\therefore L_2 \Rightarrow 7.5 \text{ cm}$$

Q.3) The kinetic energy of a simple harmonic oscillator is oscillating with angular frequency of 176 rad/sec . The frequency of this simple harmonic oscillator is ____ Hz. ($\pi = 22/7$). [JEE Main 2026]

Ans: * $\omega = 2\pi f$

$$\therefore f = \frac{\omega}{2\pi} \Rightarrow \frac{176}{2 \times \frac{22}{7}} \Rightarrow \frac{176 \times 7}{44} \Rightarrow \underline{\underline{28 \text{ Hz}}}$$

{for this simple oscillator}.

Oscillations

Q.1. A particle executes SHM with an amplitude A . At what displacement from the mean position is its speed equal to half of its maximum speed?

- A) $A/2$
- B) $A\sqrt{3}/2$
- C) $A/\sqrt{2}$
- D) $A/4$

Ans. Velocity of a particle in SHM at displacement x
 $\Rightarrow v = \omega \sqrt{A^2 - x^2}$

v_{max} at mean position ($x=0$) $\Rightarrow v_{\text{max}} = \omega A$

According to questions, $v = \frac{1}{2} v_{\text{max}}$.

On substituting,

$$\omega \sqrt{A^2 - x^2} = \frac{1}{2} \omega A$$

Squaring both sides,

$$A^2 - x^2 = \frac{A^2}{4}$$

$$x^2 = A^2 - \frac{A^2}{4} = \frac{3A^2}{4}$$

$$x = \frac{\sqrt{3}A}{2}$$

$\therefore$ B is the correct answer

Q2 Two springs of force constants k_1 and k_2 are connected in series to a mass m . If the same two springs were connected in parallel to the same mass, what is the ratio of the time period in series (T_s) to the time period in parallel (T_p)?

A) $\sqrt{k_1 k_2}$

~~B) $\frac{(k_1 + k_2)^2}{\sqrt{k_1 k_2}}$~~

B) $\frac{k_1 + k_2}{k_1 k_2}$

C) $\frac{\sqrt{(k_1 + k_2)^2}}{k_1 k_2}$

D) $\frac{k_1 + k_2}{\sqrt{k_1 k_2}}$

Ans. Time Period (T) = $2\pi \sqrt{\frac{m}{k_{eq}}}$

Series $\Rightarrow \frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$

$\Rightarrow k_s = \frac{k_1 k_2}{k_1 + k_2}$

Parallel $\Rightarrow k_p = k_1 + k_2$

Ratio,

$\frac{T_s}{T_p} = \frac{2\pi \sqrt{m/k_s}}{2\pi \sqrt{m/k_p}} = \sqrt{\frac{k_p}{k_s}}$

$\frac{T_s}{T_p} = \sqrt{\frac{k_1 + k_2}{\frac{k_1 k_2}{k_1 + k_2}}} = \sqrt{\frac{(k_1 + k_2)^2}{k_1 k_2}} = \frac{k_1 + k_2}{\sqrt{k_1 k_2}}$

$\therefore$ D is the correct answer.

Q3 In SHM, the fraction of total energy that is kinetic when the displacement is half of the amplitude is:

A) $1/4$

B) $1/2$

C) $3/4$

D) $1/3$

Ans Total Energy (E_{total}) in SHM = $\frac{1}{2} kA^2$

Potential Energy (U) at $x = \frac{A}{2}$ is:

$$U = \frac{1}{2} k \left(\frac{A}{2} \right)^2 = \frac{1}{4} \left(\frac{1}{2} kA^2 \right) = \frac{1}{4} E_{\text{total}}$$

$$E_{\text{total}} = K.E + U, \quad K.E = E_{\text{total}} - \frac{1}{4} E_{\text{total}} = \frac{3}{4} E_{\text{total}}$$

$\therefore$ Answer is C.

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OSCILLATIONS

Q.1. A cylindrical block of mass M and area of cross section A is floating in a liquid of density ρ and with its axis vertical. When depressed and released the block starts oscillating. Period of oscillations is _____.

Ans. At Equilibrium:— $m_g = \rho g A h$ [h → Submerged height]

- After displacing:— Additional Vol. submerged = Ax
- Additional Buoyant Force upward = $\rho g Ax$
- Restoring force:— $F = -\rho g Ax$ [Extra buoyant force acts upward & tries to bring block at eq^m.]

$$M \frac{d^2x}{dt^2} + \frac{\rho g Ax}{m} = 0$$

$$\Rightarrow \omega^2 = \frac{\rho g Ax}{m}$$

$$\Rightarrow \omega = \sqrt{\frac{\rho g Ax}{m}}$$

$$\Rightarrow \text{Time period} = \boxed{2\pi \sqrt{\frac{m}{\rho g Ax}}}$$

Q.2. A particle executes simple harmonic motion with an amplitude of 5cm. When the particle is at 4cm from mean position, the magnitude of its velocity in SI Units is equal to that of its acceleration. Then, Time Period in second is _____

Ans $v = \omega \sqrt{A^2 - x^2}$

At $x = 4$, $v = \omega \sqrt{5^2 - 4^2} = \underline{\underline{3\omega}}$

acceleration = $-\omega^2 x$

At $x = 4$, $a = \underline{\underline{-4\omega^2}}$

$$\therefore \omega = \frac{3}{4} \Rightarrow T = \frac{2\pi}{\omega} = \boxed{\frac{8\pi}{3}}$$

Q.3. A cylindrical plastic bottle of negligible mass is filled with 310ml of water and kept floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency. If the radius of the bottle is 2.5cm, then ω is — [$\rho_{\text{water}} = 10^3 \text{ kg/m}^3$]

Ans

$$F = -\rho g A x$$

$$a = -\frac{\rho g A x}{m}$$

$$\Rightarrow \omega^2 = \frac{\rho g A}{m} = \frac{[\rho (\pi r^2) g]}{m} = \sqrt{\frac{10^3 \pi (2.5 \times 10^{-2})^2 \times 10}{310 \times 10^{-3}}}$$

$$\approx \underline{\underline{7.95 \text{ rad/s}}}$$

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XI A

Oscillations

Q> A Simple pendulum of string length 30cm performs 20 oscillations in 10s. The length of string required for pendulum to perform 40 oscillations in same time duration is: _____ cm

Sol.

1st Simple pendulum:

20 Oscillations in 10 seconds.

$$T_1 = \frac{10}{20} = \underline{0.5s}, l_1 = 30cm$$

2nd Simple pendulum:

40 oscillations in 10s.

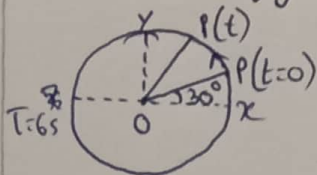
$$T_2 = \frac{10}{40} = \underline{0.25s}, l_2 = ?$$

$$\frac{L_2}{L_1} = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{0.25}{0.5}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{L_2}{30} = \frac{1}{4} \Rightarrow L_2 = \underline{7.5cm}$$

$$\boxed{l_2 = 7.5cm}$$

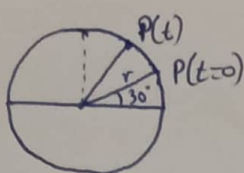
Q> For particle P revolving around centre O with radius of circular path r and angular velocity ω as shown in below figure, the projection of OP on same axis at t is:



(A) $x(t) = r \cos(\omega t - \frac{\pi}{6})$ (B) $x(t) = r \cos(\omega t)$

(C) $x(t) = r \cos(\omega t + \frac{\pi}{6})$ (D) $x(t) = r \sin(\omega t + \frac{\pi}{6})$

Sol.



angular displacement
 $\theta = \omega t$

θ from x axis

$$\theta_{total} = \omega t + \frac{\pi}{6}$$

horizontal component is along x-axis

$$\Rightarrow r \cos(\theta_{total}) = \boxed{r \cos(\omega t + \frac{\pi}{6})}$$

$$\boxed{(C) r \cos(\omega t + \frac{\pi}{6})}$$

Q2 The maximum potential energy of a block executing simple harmonic motion is 25J. 'A' is amplitude of oscillations. At $\frac{A}{2}$ kinetic energy of block is:

(A) 9.75J

(B) 37.5J

(C) 18.75J

(D) 12.5J

Sol.

$$U_{\max} = \frac{1}{2} m \omega^2 A^2 = 25J$$

$$\begin{aligned} \text{KE at } \frac{A}{2} &= \frac{1}{2} m \omega^2 \left(A^2 - \left(\frac{A}{2} \right)^2 \right) = \frac{1}{2} m \omega^2 \left(A^2 - \frac{A^2}{4} \right) \\ &= \frac{1}{2} m \omega^2 \times \frac{3A^2}{4} = \frac{3}{4} \left(\frac{1}{2} m \omega^2 A^2 \right) \end{aligned}$$

$$\text{Since, } \frac{1}{2} m \omega^2 A^2 = 25J$$

$$\text{KE at } \frac{A}{2} = \frac{3}{4} \times 25 = \frac{75}{4} = \underline{\underline{18.75J}}$$

(C) 18.75J

Oscillation.

① A particle of mass 0.5 kg executes simple harmonic motion under force $F = -50 (\text{Nm}^{-1})x$. The time period of oscillation is $\frac{x}{35} \text{ s}$. The value of x is $\frac{22 \text{ sec.}}{7}$.
(Given $\pi = \frac{22}{7}$)

Ans:-

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$\Rightarrow \frac{x}{22} = 2 \times \frac{22}{7} \times \sqrt{\frac{0.50}{50}} \Rightarrow \frac{x}{35} = \frac{44}{70} \Rightarrow x = 22$$

② The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitude 4 m , 2 ms^{-1} and 16 ms^{-2} at a certain instant. The amplitude of the motion is $\sqrt{x} \text{ m}$ where x is 17.

Ans:-

Position $\rightarrow x = A \cos(\omega t + \phi)$; ~~Velocity $\rightarrow v = A\omega \sin(\omega t + \phi)$~~

Velocity $\rightarrow -A\omega \sin(\omega t + \phi)$; $x = 4 \text{ m}$; $v = 2 \text{ ms}^{-1}$; $a = 16 \text{ ms}^{-2}$

$a = -A\omega^2 \cos(\omega t + \phi)$ | $x = A \cos(\omega t + \phi)$

$\Rightarrow 16 = A\omega^2 \cos(\omega t + \phi)$ | $\Rightarrow 4 = A \cos(\omega t + \phi)$

$\therefore A\omega^2 \cdot \frac{4}{A} = 16 \Rightarrow \omega^2 = 4 \Rightarrow \omega = 2 \text{ rad/s}$

$\therefore v = A \times 2 \times \sin(\omega t + \phi) \Rightarrow \sin(\omega t + \phi) = \frac{1}{A}$

$\cos(\omega t + \phi) = \frac{4}{A}$ | $\sin^2 \theta + \cos^2 \theta = 1$
 $\Rightarrow \left(\frac{1}{A}\right)^2 + \left(\frac{4}{A}\right)^2 = 1 \Rightarrow A^2 = 17$

$\therefore A = \sqrt{17} \text{ m}$; $x = 17$

③ A particle is doing SHM of amplitude 0.06m and time period 3.14 s . The max. velocity of the particle is 12 cm/s.

Ans:-

$$V_{\max} = A\omega \quad \left| \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{3.14} \right.$$

$$\therefore V_{\max} = A\omega = 0.06 \times \frac{2\pi}{3.14} = 0.06 \times 2$$

$$\Rightarrow V_{\max} = 0.12 \text{ m/s} \quad ; \quad V_{\max} = 0.12 \times 100 = 12 \text{ cm/s.}$$

OSCILLATIONS

(Q.) A particle executes simple harmonic motion with amplitude A and time period T . The maximum acceleration of the particle is

- (A) $\frac{A}{T^2}$ (B) $\frac{4\pi^2 A}{T^2}$ (C) $\frac{2\pi A}{T}$ (D) $\frac{A}{T}$

Solution

Acceleration in SHM $a = -\omega^2 x$

max. acceleration at $x = A$:

$$a_{\max} = \omega^2 A = \left(\frac{2\pi}{T} \right)^2$$

$$A = \frac{4\pi^2 A}{T^2} \quad \text{option (B)}$$

(Q.) A spring of force constant k is cut into two equal parts. The force constant of each part is

- (A) k (C) $2k$

- (B) $\frac{k}{2}$ (D) $\frac{k}{4}$

Solution

Force constant $k \propto \frac{1}{L}$

if length is halved $k' = 2k$

Answer (C) $2k$

(Q) A simple pendulum of length l is displaced by a small angle and released. Its time period is

(a) $2\pi\sqrt{\frac{l}{g}}$

(c) $\pi\sqrt{\frac{l}{g}}$

(b) $2\pi\sqrt{\frac{g}{l}}$

(d) $\sqrt{\frac{l}{g}}$

Solution

Time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

option (A)

Oscillation

Q.1 A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $1/1000$ of the original amplitude is close to.

Ans: $f = 5 \text{ Hz}$, $A = \frac{A}{2}$, $t_1 = 2 \text{ s}$

$$A = A_0 e^{-(b/2m)t} \quad \text{or} \quad \frac{1}{2} = e^{-(b/2m)t_1}$$

$$(b/m) = \ln 2$$

$$A = \frac{A}{1000}, \quad t_2 = ?$$

$$\frac{1}{1000} = e^{-(b/2m)t_2} \quad \text{or} \quad 10^{-3} = e^{-(b/2m)t_2}$$

$$(b/2m)t_2 = 3 \ln 10 \quad \text{or} \quad t_2 = 6 \ln 10 / \ln 2$$

$$t_2 = 20 \text{ s}$$

Q.2 A toy-car, blowing its horn, is moving with a steady speed of 5 m/s , away from a wall. An observer, towards whom the toy car is moving, is moving, is able to hear 5 beats per second. If the velocity of sound of air is 340 m/s , the frequency of the horn of the toy car is close to

Ans: $V_{\text{air}} = 340\text{ m/s}$, $V = 5\text{ m/s}$

$f_1 - f_2 = 5\text{ beats per second}$

$$f_1 = [v / (v - s)] f = [340 / (340 - 5)] f = (340 / 335) f$$

$$f_2 = [v / (v + s)] f = [340 / (340 + 5)] f = (340 / 345) f$$

$$[(340 / 335) - (340 / 345)] f = 5$$

$$f = (5 / 340) [(335 \times 345) / 10]$$

$$= 169.96\text{ Hz}$$

$$= 170\text{ Hz}$$

Q.3 The length of a simple pendulum executing SHM is increased by 21%. The percentage increase in the time period of the pendulum of increased length is?

Ans: Let the length of pendulum be 100 and 121

$$\frac{T'}{T} = \sqrt{\frac{121}{100}} = \frac{11}{10}$$

$$T' = 1.1 T$$

$$= (T' - T) / T = (1.1T - T) / T \times 100$$

$$= 10\%$$

Oscillations

Q1. A particle in SHM has equal energy 'E'. At what displacement will kinetic energy be equal to its potential energy.

Sol:

$$E = \frac{1}{2} k A^2$$

$$P.E = \frac{1}{2} k x^2, \quad K.E = E - P.E.$$

$$K.E = P.E.$$

$$E - P.E = P.E \Rightarrow E = 2 P.E.$$

$$\frac{1}{2} k A^2 = 2 \times \frac{1}{2} k x^2.$$

$$A^2 = 2x^2$$

$$x = \frac{A}{\sqrt{2}}.$$

$$\boxed{\frac{A}{\sqrt{2}}} //$$

Q2. Particle executes SHM with amplitude $A = 10 \text{ cm}$ and angular velocity frequency $\omega = 5 \text{ rad/s}$. Find velocity of the particle when it is at displacement of 6 cm from mean position.

Sol:

$$v = \omega \sqrt{A^2 - x^2}$$

$$v = 5 \sqrt{(10)^2 - (6)^2}$$

$$v = 5 \sqrt{100 - 36} = 5 \sqrt{64} = 5 \times 8 = \boxed{40 \text{ cm/s}} //$$

Q Particle executes SHM along straight line

$$\text{displacement} = x = 4 \sin \left(2t + \frac{\pi}{3} \right).$$

- (a) amplitude (b) Angular frequency (c) time period
(d) max velocity.

Sol:

(a) $\Rightarrow$ 4 cm

(b) $\Rightarrow$ 2 rad

(c) $\Rightarrow$ Time period $\Rightarrow T = \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{2} = \pi \text{ s}$$

$\Rightarrow$ $\pi \text{ s}$

(d) max velocity

$$U_{\text{max}} = \omega A = 2 \times 4 = 8 \text{ cm/s}$$

8 cm

- Q. A particle executes simple harmonic motion with an amplitude of 4 cm. At mean position, velocity of particle is 10 cm/s. The distance of the particle from mean position when its speed becomes 5 cm/s is $\sqrt{a}$ cm where $a = \underline{\hspace{1cm}}$. [JEE Mains 2024]

Sol: $v_{\text{mean pos}} = A\omega \Rightarrow 10 = 4\omega$

$$\omega = 5/2$$

$$v = \omega \sqrt{A^2 - x^2}$$

$$5 = \frac{5}{2} \sqrt{4^2 - x^2} \Rightarrow x^2 = 16 - 4$$
$$x = \underline{\underline{\sqrt{12} \text{ cm}}}$$

- Q. A particle executes SHM of amplitude A . The distance from the mean position when its $KE = PE$ is: [JEE Main 2023]

$$KE = PE$$

$$\frac{1}{2} M \omega^2 (A^2 - x^2) = \frac{1}{2} M \omega^2 x^2$$

$$(A^2 - x^2) = x^2$$

$$2x^2 = A^2$$

$$x = \pm A/\sqrt{2}$$

- Q. 2 massless springs with the spring constants 2 k & 9 k carry 50 g & 100 g masses at their free ends. These 2 masses oscillate vertically such that their maximum velocities are equal then the ratio of their respective amplitudes will be: [JEE Main 2022]

Sol: $\omega_1 A_1 = \omega_2 A_2$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1}$$

$$= \sqrt{\frac{k_2}{m_2}} \times \sqrt{\frac{m_1}{k_1}} = \sqrt{\frac{9k}{100} \times \frac{50}{2k}}$$

$$= \underline{\underline{3/2}}$$

Q.) The ' λ ' of oscillation of mass ' m '
~~is~~ suspended by a spring ' ν_1 '. If length
 of the spring is cut to half, the same mass
 oscillates with frequency ' ν_2 '. The value of
 ν_2/ν_1 is —

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\sqrt{3}$

Ans: $\nu = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$

$$K \propto \frac{1}{L}$$

$$L' = \frac{L}{2}$$

$$K' = 2K$$

$$\nu_2 = \sqrt{2} \nu_1$$

$$\frac{\nu_2}{\nu_1} = \sqrt{2}$$

Q.) A particle executing SHM. Its Amplitude A and time period is 5 sec. The time required to move from $x = A$ to $x = \frac{A}{\sqrt{2}}$ is $\text{--- sec} \times 10^{-2}$ sec

Ans

$$x = A \cos \omega t$$

$$\frac{A}{\sqrt{2}} = A \cos \omega t$$

$$t = \frac{T}{8}$$

$$t = \frac{5}{8} = \boxed{0.625 \approx 6.25} \times 10^{-2}$$

Q.) A particle of $m = 0.5 \text{ kg}$ executes SHM under $F = -50(Nm^{-1})x$. The T of oscillation is $\frac{x}{35}$ s.

The value of x is --- ($\pi = \frac{22}{7}$)

Ans:

$$m = 0.5 \text{ kg}$$

$$F = -50x$$

$$a = -100x$$

$$\omega^2 = 100, \omega = 10$$

$$T = \frac{2\pi}{\omega} = \frac{\pi}{5} = \frac{22}{7 \times 5} = \frac{22}{35}$$

$$\frac{x}{35} = \frac{22}{35}, \quad \boxed{x = 22}$$

SHM

Q. A particle undergoing simple harmonic motion has time-dependent displacement given by $x(t) = A \sin\left(\frac{\pi t}{90}\right)$. The ratio of kinetic to the potential energy of this particle at $t = 210$ s will be

$$\Rightarrow KE = \frac{1}{2} m A^2 \omega^2 \cos^2 \omega t$$

$$PE = \frac{1}{2} m A^2 \omega^2 \sin^2 \omega t$$

$$\frac{KE}{PE} = r = \frac{\cos^2 \omega t}{\sin^2 \omega t} = \cot^2 \omega t$$

$$\omega = \frac{\pi}{90} ; t = 210 \text{ s}$$

$$r = \cot^2 \left(\frac{\pi}{90} \right) 210$$

$$= \cot^2 \left(\frac{7\pi}{3} \right)$$

$$= \cot^2 \left(2\pi + \frac{\pi}{3} \right)$$

we know,

$$\cot(2n\pi + \theta) = \cot \theta$$

$$\therefore r = \cot^2(\pi/3)$$

$$= \left(\frac{1}{\sqrt{3}} \right)^2$$

$$\boxed{r = \frac{1}{3}} \rightarrow \text{Ans.}$$

Q. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $1/1000$ of the original amplitude is close to

$$\Rightarrow \text{Frequency} = 5 \text{ Hz}$$

$$A = \frac{A}{2}, t = 2 \text{ s}$$

$$A = A_0 e^{-(b/2m)t} \quad \frac{1}{2} = e^{-(b/2m)2}$$

$$b/m = \ln 2$$

$$A = \frac{A}{1000}, t_2 = ?$$

$$\frac{1}{1000} = e^{-(b/2m)t_2}$$

$$10^{-3} = e^{-(b/2m)t_2}$$

$$(b/2m)t_2 = 3 \ln 10$$

$$\boxed{t_2 = 20 \text{ s}} \rightarrow \underline{\text{Ans}}$$

Q. A object of mass 0.2 kg executes a simple harmonic oscillation along the x -axis with a frequency of $(25/\pi)$

At position $x = 0.04$, the object has a kinetic energy of 0.5 J and potential energy 0.4 J . Amplitude is.

$$\Rightarrow m = 0.2 \text{ kg}; \omega = \frac{25}{\pi}; KE = 0.5; PE = 0.4$$

$$\text{Total energy} = 0.9 \text{ J}$$

$$T.E = \frac{1}{2} m \omega^2 a^2 \rightarrow 0.9 = \frac{1}{2} (0.2 \times 4 \pi^2 \times (\frac{25}{\pi}) (\frac{25}{\pi}) \times a^2)$$

$$\boxed{a = 0.06 \text{ m}} \rightarrow \underline{\text{Ans}}$$

Q1) The length of sonometer wire between its fixed ends is 110 cm. Where two bridges should be placed in between the ends so as to divide the wire into 3 segments whose fundamental frequencies are in the ratio 1:2:3?

$$A) L_1 + L_2 + L_3 = 110 \text{ cm}$$

$$f_1 : f_2 : f_3 = 1 : 2 : 3$$

$$f_1 = f$$

$$f_2 = 2f$$

$$f_3 = 3f$$

Tension remains same throughout wire.

$$f_1 L_1 = f_2 L_2 = f_3 L_3$$

$$f L_1 = 2f L_2 = 3f L_3$$

$$L_1 = 2L_2 = 3L_3$$

$$\text{if } L_3 = x, L_2 = \frac{3}{2}x, L_1 = 3x$$

$$L_1 + L_2 + L_3 = 3x + \frac{3}{2}x + x = 110 \text{ cm}$$

$$11x = 220 \text{ cm}$$

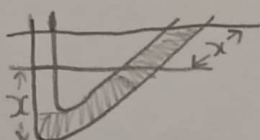
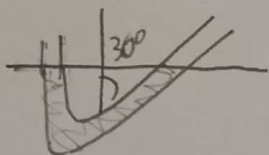
$$L_3 = x = 20 \text{ cm}$$

$$L_2 = \frac{3}{2}x = 30 \text{ cm}$$

$$L_1 = 3x = 60 \text{ cm}$$

$\therefore$ The bridges should be placed at 54.54 cm from zero end and 18.18 cm from the end.

Q2) Determine period of oscillation of mercury of mass $m = 200 \text{ g}$, poured into bent tube whose right arm forms an 30° angle with vertical. Cross sectional area, $S = 0.5 \text{ cm}^2$. Ignore viscosity of mercury.



A) Difference in level of two tubes
 $= x + x \cos \theta = x(1 + \cos \theta)$

Change in pressure $\Delta P = x(1 + \cos \theta) \rho g$

$$F = ma = -(x(1 + \cos \theta) \rho g) S$$

$$a = \frac{(1 + \cos \theta) \rho g S}{m} = -\omega^2 x, \quad \omega^2 = \frac{(1 + \cos \theta) \rho g S}{m}$$

In the form of equation of SHM.,

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{(1 + \cos \theta) \rho g S}}$$

$$T = 2 \times 3.14 \sqrt{\frac{0.2}{(1 + \cos 30^\circ)(13.6 \times 10^3)(9.8)(5 \times 10^{-5})}} = 0.8 \text{ seconds} //$$

Q3) Find time period of motion of a particle shown in figure. Neglect the small effect of bend near the bottom.

A) The time of motion for one cycle

$$= 2(t_{AB} + t_{BC})$$

$$t_{AB} = \sqrt{\frac{2AB}{g \sin \theta}}$$

$$t_{BC} = \sqrt{\frac{2BC}{g \sin \beta}}$$

$$AB = h \csc \theta$$

$$BC = h \csc \beta$$

$$H = 10 \text{ cm}, \theta = 45^\circ, \beta = 60^\circ$$

Applying values

$$T = \frac{2\sqrt{2}}{\sqrt{g}} \left(\sqrt{\frac{AB}{\sin \theta}} + \sqrt{\frac{BC}{\sin \beta}} \right)$$

$$= 0.726 \text{ seconds} //$$

